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# STRESSES DUE TO INTERIOR COOLING

BY

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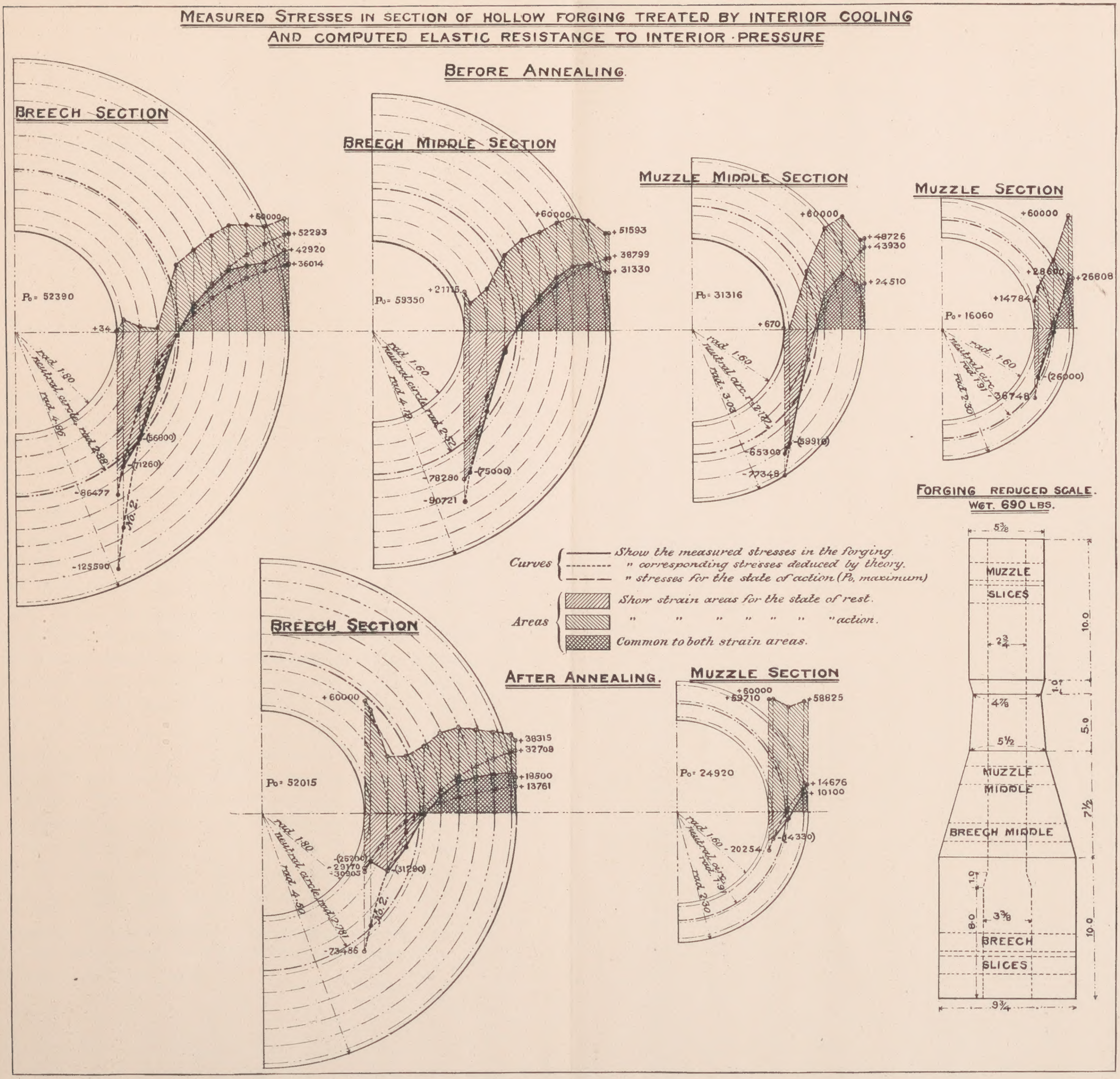


STRESSES IN GUN FORGINGS.

# STEEL CYLINDERS FOR GUN CONSTRUCTION—STRESSES DUE TO INTERIOR COOLING.

BY

ROGERS BIRNIE.

[Read before the Society May 11, 1895.]

This experiment is the first important step taken by the Ordnance Department of the United States Army to investigate the merits of making cannon from a single steel forging with initial tension produced by interior cooling. The experiments of similar import with reference to built-up or hooped steel guns which were made by this department in 1884–'6 established on a very firm basis the manufacture in this country of that description of gun, and it is not improbable that the present investigation may lead to equally important results for the single forging construction.

The inception of this work is due to Captain Frank Hobbs, of the Ordnance Department, and the experimental forging was furnished, through his instrumentality, by the Bethlehem Iron Co. The subject has received attention in other places, particularly in the interesting work of General Nicholas Kalakoutsky,* of the Russian artillery, but up to this time guns have not been made upon the plan proposed. At Le Creusot, France, interior cooling has, however, been used to improve the condition of cylinders used in built-up guns. As to the treatment of the forging, it may suffice to say that its preparation was similar to that of a forging intended for a hooped gun, by the usual methods of casting,

* Kalakoutsky (General Nicholas). Investigations into the internal stresses of cast iron and steel. London, George Reveirs, 1888.

forging, annealing, oil-tempering and annealing. In the final annealing, while still in the annealing furnace and uniformly heated to redness, water was passed into and through the bore of the forging until it was cool enough to handle.

The several circular sections shown in the drawings, each about 0.5 of an inch thick, were then cut from different parts of the forging to ascertain the strains in concentric elementary cylinders of which the section may be conceived to be composed. Each section was marked to be divided into a number of circular rings about 0.15 of an inch in radial thickness. Before cutting out these rings datum points were marked on the face of each to measure two diameters at right angles. The rings were removed consecutively and measurements of the diameters made at each stage of the operation. The change in diameter of a ring on being released from the section is taken as a measure of the circumferential strain or stress to which it was subjected in the forging. A ring which expands on being released was evidently under circumferential compression in the forging, and one which contracts was under tension. The datum points for the curves of initial tension shown in the figures are derived from the difference in the original diameters of the rings and their diameters after release. As seen in these curves, the compression is greatest at or near the surface of the bore, whence it gradually decreases to zero at the neutral point. At this point the strains of tension begin and increase gradually toward the exterior of the cylinder. The strains of compression and extension are in equilibrium.

Duplicate sections from the breech and muzzle ends of the forging are illustrated, the originals being taken directly after the treatment by interior cooling and the duplicates when the parts of the forging to which they belonged had been subjected to partial annealing. The object of this treatment was to show how the strains originally produced could be controlled and ameliorated, if necessary, by annealing a forging after interior cooling.

The accuracy of the results is of course dependent upon the measurements of diameters. Of this, however, there is every indication that proper care and skill were exercised. The measurements were made with a micrometer scale and read to the fractional part of $\frac{1}{10000}$ of an inch. The sensitiveness of the results is such that an error of $\frac{1}{10000}$ of an inch in the reading of an average diameter (five inches) corresponds to 600 pounds per square inch in the expressed stress of tension or compression.

*Deductions from the Application of the Formulas for Gun Construction.*—These and other similar experiments show the favorable condition of strains produced in a hollow forging by interior cooling. The strains are analogous to those produced by shrinkage in the built-up construction and serve the same purpose. The present experiments are particularly instructive in that they deal with a hollow forging of varying thickness of wall, with sectional dimensions corresponding to the service field gun.

The strains directly produced by the treatment are found to be more intense than is necessary and show the desirability of an amelioration of that treatment in future operations; but the strains left after annealing the treated forging are moderate and satisfactory. The elastic resistance of the sections, both before and after annealing, is shown to be superior to that of corresponding sections of the built-up field gun. This, however, is in part attributable to higher qualities of metal.

The physical qualities of the forging, determined from tensile-test specimens taken from it after treatment, are as follows:

| | *Breech end.* | *Muzzle end.* |
|---|---|---|
| Elastic limit, pounds per square inch.......... | 68,000 | 75,500 |
| Tensile strength, " " " " .......... | 126,500 | 128,400 |
| Ultimate extension, per cent.................. | 9.50 | 11.625 |
| Reduction of area, " " .................. | 12.14 | 16.35 |

For present purposes the elastic limit for extension will be taken at a reduced value, $\theta = 60,000$, and the elastic limit for compression will be defined in each section by the actual

measurements made, the highest being $\rho = 78{,}280$ at the bore of the breech middle section. It will be understood that the stated measured stresses correspond to the measured strains per inch for a modulus of elasticity, $E = 30{,}000{,}000$ pounds. For example, the value of $\rho$ just stated is derived as follows:

$$\rho = \frac{0.00835 \text{ (strain)}}{3.2 \quad \text{(diameter)}} 30{,}000{,}000 = 78{,}280 \text{ (stress)}. \qquad (1)$$

The sections taken for examination are the breech, breech middle, muzzle middle, and muzzle before annealing, and the breech and muzzle after annealing. In the dimensions of these sections we have nearly the counterpart of four principal cross sections of the 3.2-inch field gun.

The principal objects of discussion will be—

1. To compare the measured stresses in the forging after treatment with those anticipated by theory and required to make the resistance to interior pressure a maximum. This will show the degree of uniformity in the actual stresses and how nearly they conform to the requirements of the law for maximum resistance.

2. Taking the actual stresses as measured in each section, to determine the elastic resistance of the section to interior pressure. This, while admitting every irregularity of the stresses or strains induced by the treatment, will give a final measure of its efficacy.

The formulas* to be applied, which are fundamentally the same as those for the built-up construction, relate to a gun or cylinder made of a single piece, with initial tension produced by interior cooling.

$$\theta_u = \tfrac{2}{3} \frac{P}{\left(\frac{R_1}{R_0}\right)^{\frac{2}{3}} - 1} = a\,P \qquad (2)$$

$$P_u = \frac{3\,(R_1^2 - R_0^2)\,\rho}{(4\,R_1^2 + 2\,R_0^2) - 3\,(R_1^2 - R_0^2)\,a} \qquad (3)$$

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*For these formulas *see* Gun Making, Appendix B, Military Service Institution Monograph, 1888.

In these equations $P_u$ is the interior pressure per square inch which, if applied, would produce an uniform stress, $\theta_u$, in the whole thickness of the wall of the cylinder, depending only upon the dimensions of the cylinder and the initial compression $\rho$.

$$\theta = \left[\frac{2\,R_0^2}{3\,(R_1^2 - R_0^2)} + \frac{4\,R_1^2\,R_0^2}{3r^2\,(R_1^2 - R_0^2)}\right] P \qquad (4)$$

In this $\theta$ is the *increase* of stress caused at radius $r$ by the application of any interior pressure, $P$, within the limit of elasticity of the cylinder.

$$\rho = -\left[\frac{2\,R_0^2}{3\,(R_1^2 - R_0^2)} + \frac{4\,R_1^2\,R_0^2}{3r^2\,(R_1^2 - R_0^2)}\right] P_u + \theta_u \qquad (5)$$

In this $\rho$ is the *relief* of stress at radius $r$, with given values of $P_u$ and $\theta_u$, under the assumption that the pressure $P_u$ has been applied and is withdrawn.

Replacing $\theta_u$ in (5) by its value as expressed in (2) and designating by $\rho_1$ a stress at radius $r_1$, the ratio of the stresses at two different points in the wall may be expressed by

$$\rho = \left[\frac{\dfrac{1}{\left(\dfrac{R_1}{R_0}\right)^{\frac{2}{3}} - 1} - \dfrac{R_0^2}{R_1^2 - R_0^2} - \dfrac{2\,R_1^2\,R_0^2}{(R_1^2 - R_0^2)\,r^2}}{\dfrac{1}{\left(\dfrac{R_1}{R_0}\right)^{\frac{2}{3}} - 1} - \dfrac{R_0^2}{R_1^2 - R_0^2} - \dfrac{2\,R_1^2\,R_0^2}{(R_1^2 - R_0^2)\,r_1^2}}\right] \rho_1 \qquad (6)$$

From this the stress $\rho$ at a given radius, $r$, may be determined when $\rho_1$ for the radius $r_1$ is given. The symbols $\rho$ and $\rho_1$ in this equation may express either compression or tension.

The radius of the circle on which should be found the neutral point of every curve of stress incident to the system at rest is found from the equation

$$r = R_1 \sqrt{\frac{2\left(\dfrac{R_0}{R_1}\right)^{\frac{2}{3}} - 2}{\left(\dfrac{R_1}{R_0}\right)^2 - \left(\dfrac{R_1}{R_0}\right)^{\frac{2}{3}}}} \qquad (7)$$

Additional formulas deduced from those given in Appendix B, "Gun Making," can be given to determine, first, the changes in the stresses at given radii due to reaming out or enlarging the bore of a cylinder under initial tension; second, the changes due to turning off or reducing the exterior of the cylinder. Such formulas would be useful in the practical working of this method, but it will be sufficient to state here that any reduction of the thickness of wall should cause a lowering of the initial strains or stresses. This result has not been uniformly shown in the present experiments. Exceptions may be noted in the earlier stages of dismantling the breech and the breech middle sections. In these cases it would appear that the metal near the surface of the bore was overcompressed by the treatment, and local strains were produced which became manifest when a part of the metal was removed.

To explain the application of the formulas we may take, for example, the breech section before annealing.

*Stress Curves for the State of Rest.*—Taking the measured compression, 71,260 pounds, on the diameter, 3.81 inches, as a basis for constructing the "deduced" curve of stresses, we first find the corresponding compression at the surface of the bore from equation (6), in which

$$r = R_0 = 1.8,\ r_1 = 1.905,\ R_1 = 4.86,\ \rho_1 = 71260.$$

Then:

$$\rho_0 = \frac{1.0647 - 0.15898 - 2.0695}{1.0647 - 0.15898 - 1.5446}\,\rho_1 = \frac{1.41228}{1.16378} \times 71260 = 86477 \text{ lbs.}$$

Next, applying equations (3) and (2) with $\rho_0$ given, we find:

$$P_u = \frac{3 \times 20.38}{100.96 - 43.41}\,\rho_0 = \frac{61.14}{57.55} \times 86477 = 91870 \text{ pounds.}$$

$$\theta_u = a\,P_u = 0.70997 \times 91870 = 65225 \text{ pounds.}$$

These latter values express the theoretical condition, assumed only for auxiliary purposes, that, having a com-

pression of 71,260 pounds at the intermediate radius, 1.905 inches, an applied interior pressure of 91,870 pounds would produce throughout the whole wall of the cylinder an uniform tension of 65,225 pounds per square inch.

The remaining points of the deduced curve of stresses for the state of rest are now derived by equation (5). Having $\theta_u = 65{,}225$ and $P_u = 91{,}870$, we find:

$$\rho = 55488 - \frac{459970}{r^2}$$

in which, by substituting the several values of $r$, there results:

$$\left.\begin{array}{l} D_0 = 3.6,\ R_0 = 1.8:\ \rho = 55488 - 141965 = -86477 \\ d = 3.81,\ r = 1.905:\ \rho = 55488 - 126750 = -71260 \end{array}\right\} \text{(Proof.)}$$

$$d = 4.41,\ r = 2.205:\ \rho = 55488 - 94604 = -39116$$

&c., &c.,

as given in table A and shown on the accompanying plate 6.

The "deduced" stress curves for the state of rest in the remaining sections considered are derived in a similar manner. In each case the measured stress which is taken as a basis and so forms a common point on both the measured and deduced curves of stress is designated (see table and plate 6) by figures in parentheses, as, for example (75,000) in the breech middle section, (59,910) in the muzzle middle section, and so on.

*Resistance to Interior Pressure.*—The limit of elastic resistance of the metal under extension will be taken as before stated, $\theta = 60{,}000$. The value of $P_0$ will then depend upon the condition that this limit shall not be exceeded at any point. By a comparison of the measured and deduced stress curves for the state of rest, or, if need be, by a preliminary computation, the most dangerous measured stress—that is to say, the one which, under the action of an interior pressure, would be the first to reach the limit, $\theta = 60{,}000$, can readily be selected. Consequently the points which must be taken upon which to base the value of $P_0$ for the several sections are selected and designated (see table) by

underlining the critical measured stress; for example, 42920 on the diameter 9.54 inches in the breech section, and so on. In each case it is seen from the table that the corresponding stress in action is 60,000 pounds, while the stresses on other diameters are less than this; hence the condition is fulfilled.

Taking again the breech section (second stage) before annealing as an example of the method of computation, the elastic resistance of the section and the stresses on given diameters for the state of action are derived as follows:

The measured stress which in this section will first reach the limit, 60,000, is $\theta = 42,920$ on the diameter, 9.54 inches. The increase of stress allowable on this diameter in passing from the state of rest to action is therefore:

$$\theta = 60000 - 42920 = 17080 \text{ pounds.}$$

The corresponding value of $P_0$ is then found from (4), with $r = 4.77$.

$$\theta = 17080 = 0.32602\, P_0 \quad \therefore \quad P_0 = 52390 \text{ pounds.}$$

The interior pressure being thus determined, equation (4) is further applied to determine the *increase* of stress at other given radii. For this purpose it is convenient to reduce it to the form by substituting known values:

$$\theta = 5553 + \frac{262300}{r^2}$$

in which, by substituting the several values of $r$, we obtain the increase of stress for that radius, and, taking the algebraic sum of this result and the measured stress at the same point (at rest), we have finally the stress pertaining to the applied interior pressure, $P_0 = 52,390$ pounds. Thus:

| | | Increase. | Measured. | $\theta$ (action). |
|---|---|---|---|---|
| $d = 3.81,\ r = 1.905$: | $\theta = 5553 + 72280$ | $= 77833$ | $- 71260$ | $= + 6573$ pounds. |
| $d = 4.41,\ r = 2.205$: | $\theta = 5553 + 53949$ | $= 59502$ | $- 56800$ | $= + 2702$ " |
| * | * | * | * | * * * |
| $d = 9.54,\ r = 4.77$: | $\theta = 5553 + 11528$ | $= 17080$ | $+ 42920$ | $= + 60000$ " |

(Proof.)

as given in table A and shown on the accompanying plate 6.

The radius of the neutral circle of stress for each section has been computed by (7) and is noted in the table.

### Table A.

*Measured and Deduced Stresses for the State of Rest and Action.*

BEFORE ANNEALING.

Breech section, second stage. Original diameters, $\begin{cases} D_0 = 3.34 \text{ inches.} \\ D_1 = 9.77 \text{ inches.} \end{cases}$

| Diameters. | State of rest—stresses. Measured. | State of rest—stresses. Deduced. | State of action. Pressure. | State of action. Stresses. | |
|---|---|---|---|---|---|
| *Inches.* | *Pounds per sq. inch* | *Pounds per sq. inch* | *Pounds per sq. inch.* | *Pounds per sq. inch.* | |
| Bore, 3.6 | ................. | — 86,477 | 52,390 (elastic resistance). | + 34 | Thickness of section in calibers, 0.85. |
| 3.81 | — 71,260 | —(71,260) | | + 6,573 | |
| 4.41 | — 56,800 | — 39,116 | | + 2,702 | |
| 5.05 | — 30,000 | — 16,658 | | + 1,694 | |
| 5.70 | — 2,370 | — 1,142 | | + 35,477 | |
| * | | | | | |
| 6.30 | + 14,290 | + 9,132 | | + 46,278 | |
| 6.95 | + 23,960 | + 17,396 | | + 51,235 | |
| 7.57 | + 32,900 | + 23,382 | | + 56,762 | |
| 8.20 | + 35,400 | + 28,125 | | + 56,557 | |
| 8.82 | + 36,700 | + 31,837 | | + 55,740 | |
| 9.54 | + 42,900 | + 35,272 | | + 60,000 | Elastic limit. |
| Exterior, 9.72 | ................. | + 36,014 | | ................. | |

* Neutral point, $d = 5.76$.

Breech middle section, second stage. Original diameters, $\begin{cases} D_0 = 2.80 \text{ inches.} \\ D_1 = 8.68 \text{ inches.} \end{cases}$

| | | | | | |
|---|---|---|---|---|---|
| Bore, 3.20 | — 78,280 | — 90,721 | 59,350 (elastic resistance). | + 21,115 | Thickness of section in calibers, 0.81. |
| 3.38 | — 75,000 | —(75,000) | | + 14,789 | |
| 4.02 | — 42,910 | — 35,169 | | + 22,595 | |
| 4.64 | — 10,670 | — 11,209 | | + 40,146 | |
| * | | | | | |
| 5.26 | + 7,410 | + 4,791 | | + 48,450 | |
| 5.90 | + 18,560 | + 16,304 | | + 52,566 | |
| 6.50 | + 28,380 | + 24,159 | | + 57,587 | |
| 7.08 | + 34,320 | + 29,934 | | + 60,000 | Elastic limit. |
| 7.65 | + 35,490 | + 34,378 | | + 58,454 | |
| 8.24 | + 30,760 | + 38,041 | | + 51,485 | |
| Exterior, 8.38 | + 31,330 | + 38,799 | | + 51,593 | |

* Neutral point, $d = 5.04$.

Muzzle middle section, second stage. Original diameters, $\begin{cases} D_0 = 2.80 \text{ inches.} \\ D_1 = 6.44 \text{ inches.} \end{cases}$

| | | | | | |
|---|---|---|---|---|---|
| Bore, 3.20 | — 65,300 | — 77,348 | 31,316 (elastic resistance). | + 670 | Thickness of section in calibers, 0.45. |
| 3.38 | — 59,910 | —(59,910) | | + 56 | |
| 4.02 | — 13,430 | — 15,737 | | + 31,328 | |
| * | | | | | |
| 4.64 | + 19,070 | + 10,838 | | + 54,679 | |
| 5.26 | + 30,500 | + 28,583 | | + 60,000 | Elastic limit. |
| 5.90 | + 22,630 | + 41,352 | | + 47,734 | |
| Exterior, 6.06 | + 24,510 | + 43,930 | | + 48,726 | |

* Neutral point, $d = 4.354$.

TABLE A—Continued.

BEFORE ANNEALING.

Muzzle section, third stage. Original diameters, $\begin{cases} D_0 = 2.80 \text{ inches.} \\ D_1 = 5.39 \text{ inches.} \end{cases}$

| Diameters. | State of rest—stresses. Measured. | State of rest—stresses. Deduced. | State of action. Pressure. | State of action. Stresses. | |
|---|---|---|---|---|---|
| Inches. | Pounds per sq. inch. | Pounds per sq. inch. | Pounds per sq. inch | Pounds per sq inch. | |
| Bore, 3.2 | ........ | — 36,748 | 16,060 (elastic resistance). | + 14,784 | Thickness of section in calibers, 0.22. |
| 3.35 | — 26,000 | —(26,000) | | + 21,900 | |
| * | | | | | |
| 3 90 | — 1,925 | + 3,378 | | + 36,049 | |
| 4.46 | + 28,600 | + 22,836 | | + 60,000 | Elastic limit. |
| Exterior, 4.60 | ........ | + 26,808 | | ........ | |

* Neutral point, $d = 3.82$.

AFTER ANNEALING.

Breech section, third stage. Original diameters, $\begin{cases} D_0 = 3.34 \text{ inches.} \\ D_1 = 9.77 \text{ inches.} \end{cases}$

| Diameters. | Measured. | Deduced. | Pressure. | Stresses. | |
|---|---|---|---|---|---|
| Bore, 3.6 | — 29,170 | — 30,905 | 52,015 (elastic resistance). | + 60,000 | Elastic limit. |
| 3.81 | — 25,20 | —(25,200) | | + 55,119 | |
| 4.41 | — 31,290 | — 13,164 | | + 30,330 | Thickness of section in calibers, 0.75. |
| 5.05 | — 18,120 | — 4,752 | | + 30,443 | |
| * | | | | | |
| 5.70 | — 790 | + 1,058 | | + 38,749 | |
| 6.30 | + 9,048 | + 4,906 | | + 42,613 | |
| 6.95 | + 16,620 | + 8,002 | | + 45,378 | |
| 7 57 | + 19,220 | + 10,243 | | + 44,497 | |
| 8.20 | + 20,600 | + 12,020 | | + 43,119 | |
| 8.82 | + 21,500 | + 13,410 | | + 41,860 | |
| Exterior, 9.00 | + 18,500 | + 13,761 | | + 38,315 | |

* Neutral point, $d = 5\ 562$.

Muzzle section, third stage. Original diameters, $\begin{cases} D_0 = 2.80 \text{ inches.} \\ D_1 = 5.38 \text{ inches.} \end{cases}$

| Diameters. | Measured. | Deduced. | Pressure. | Stresses. | |
|---|---|---|---|---|---|
| Bore, 3.2 | ........ | — 20,254 | 24,920 (elastic resistance). | + 59,710 | |
| 3.35 | — 14,330 | —(14,330) | | + 60,000 | Elastic limit. |
| * | | | | | |
| 3.90 | — 3,460 | + 1,861 | | + 55,467 | Thickness of section in calibers, 0.22. |
| 4.46 | + 10,100 | + 12,586 | | + 58,825 | |
| Exterior, 4.60 | ........ | + 14,676 | | ........ | |

* Neutral point, $d = 3.82$.

*The Possible Maximum Resistance of the Sections.*—Suppose the initial tension curve to be such as is required by theory to give a maximum resistance, several propositions may be stated :

1st. The point of critical strain may always be taken at the surface of the bore where the compression at rest or the

extension in action cannot exceed the elastic limit of the metal.

2d. To make the resistance to interior pressure a maximum in any cylinder, the state of initial tension should be such that when the pressure acts from within the whole thickness of metal in the wall should be, as nearly as practicable, uniformly strained to the elastic limit of the metal.

3d. If $\rho$ and $\theta$ be taken equal and the compression of bore carried to the limit $\rho$, there is but one thickness of cylinder (0.65 caliber, nearly)* for which a condition of uniform strain in action equal to the elastic limit of the metal can be attained.

4th. For cylinders of greater thickness than 0.65 caliber a state of uniform strain in the wall will be reached in action and passed before the elastic limit of the metal is attained, and with increasing pressure this limit will be fully reached only at the surface of the bore, thus determining the limit of pressure. For such cylinders the best conditions of resistance will be obtained by utilizing the full limit of compression of the metal in the initial tension.

5th. But for cylinders of less thickness than 0.65 caliber a state of uniform strain in action equal to the elastic limit of the metal can be attained with a compression of bore less than the limit $\rho$. The thinner the cylinder the less should be the initial compression imposed. It follows that the possible maximum resistance of such cylinders will be obtained by adjusting the initial compression within limits. If the full limit of initial compression were given, the elastic limit of the metal would be reached in action at the exterior of the cylinder sooner than at the bore.

6th. As a consequence, also, of the preceding, the resistance of cylinders of less thickness than 0.65 caliber, treated by interior cooling, should be directly proportional to the thickness. This treatment gives the means of imparting the greatest resistance so far known to such cylinders.

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*See Appendix B, "Gun Making" and "Modern Gun Construction and Breech Mechanism," Congress of Engineers, 1893.

7th. When one point of the initial tension curve is given (either an assigned or measured value of $\rho$) the curve will be fixed, as has been illustrated in the examples worked out. It is important to observe that this curve as defined and laid down is, barring the presence of local strains in the metal, that which should be naturally formed under the conditions of equilibrium between the positive and negative strains in the wall of the cylinder at rest. This equilibrium must exist, and the curve as defined fulfills this condition, since it is dependent upon it. The straight line, representing the state of uniform strain in action, used in every case as the datum line for the initial tension curve, is dependent upon the condition of equilibrium between the pressure and the elastic strains in the metal. That line, however, is the initial tension curve itself in a particular position. On the withdrawal (supposed) of the force $P$ the right line falls to the position of the initial tension curve, having lost nothing of its property to express the equilibrium of the forces which cause it to exist. This being, then, the only curve which can be formed under the circumstances, and since the value of $\rho$ cannot exceed the given elastic limit, it may be seen why in cylinders thicker than 0.65 caliber the conditions of maximum pressure and uniform strain to the elastic limit of the metal in action cannot exist together. An initial tension curve, starting with the limit $\rho$ at the bore and having higher strains than the natural curve toward the exterior, might be laid down which would become a straight line with $P$ increased sufficiently to stretch the bore to the elastic limit, but such an initial tension curve is not attainable in practice.

To compute the possible maximum resistance of the present sections of cylinders we will take, as before, the conservative limits, $\theta = 60{,}000 = \rho$. The breech (after annealing) and the breech middle sections are respectively 0.75 and 0.81 caliber in thickness. Their maximum resistance will therefore depend upon the assumption that the bore is initially compressed to the limit $\rho$ at rest and extended to the

limit $\theta$ in action, and its value will be found from equation (1) (Appendix B, "Gun Making"); whence

$$\text{Breech:} \quad P = \frac{3\,(R_1^2 - R_0^2)}{4\,R_1^2 + 2\,R_0^2}(\rho + \theta) = \frac{51.03}{87.48} \times 120000 =$$

70000 pounds per square inch.

$$\text{Breech middle:} \quad P = \frac{44.99}{75.34} \times 120000 = 71652 \text{ pounds per square inch.}$$

The muzzle middle and muzzle sections are less than 0.65 caliber thickness, being respectively 0.45 and 0.22 caliber. The desired initial compression, or that value of $\rho$, in terms of the limit $\theta$, which will cause the wall to be uniformly strained to 60,000 pounds per square inch when the limit of interior pressure is reached, is found by combining equations (1) and (2) of the work cited; whence

$$\rho = \left[\frac{4\,R_1^2 + 2\,R_0^2}{3\,(R_1^2 - R_0^2)a} - 1\right]\theta \tag{8}$$

From which we find:

Muzzle middle: $\rho = 0.6769 \times 60000 = 40614$
Muzzle: $\rho = 0.3174 \times 60000 = 19046$

These reduced values of $\rho$ being given, the maximum resistance will be found by applying equation (1) as for the other sections. Then

$$\text{Muzzle middle:} \quad P = \frac{19.8627}{41.8436}(40614 + 60000) = 47650 \text{ pounds per square inch.}$$

$$\text{Muzzle:} \quad P = \frac{8.19}{26.28}(19046 + 60000) = 24634 \text{ pounds per square inch.}$$

The value 24,634 for the muzzle section is less than 24,920, which was computed, from the most dangerous one of the actually measured stresses, to be the resistance after annealing. This slight discrepancy is due to the irregularities of

the curve of measured stresses and that there is no measured stress for the exterior surface, where, as every indication points, the critical strain should actually be located. It will be observed that the curve of deduced stress at rest indicates a stress of 14,676 pounds at the exterior surface. Taking the latter to govern the resistance, we find, from equation (4), $P_0 = 24{,}167$. It may be said, therefore, that the probable resistance, based upon an extension limit of 60,000 pounds as taken, lies between 24,167 and 24,634 (theoretical maximum) instead of being 24,920.

Similar conditions are also present in the breech and muzzle sections before annealing, and the values given for them in table A and drawing should probably be reduced in the same proportion, or about 3 per cent. This reduction is made in the following table, which gives a comparison of the resistance of these sections and those of corresponding dimensions in the built-up field gun as now made. It must be observed, however, that in the field gun the tube is not hooped in the two forward sections.

*Comparative Resistance of Initial Tension Cylinders and the 3.2-inch Built-up Field Gun.*

| Resistance, estimated. | | | Breech section. | Breech mid-section. | Muzzle mid-section. | Muzzle section. |
|---|---|---|---|---|---|---|
| | | | *Pounds per sq. inch.* | *Pounds per sq. inch.* | *Pounds per sq. inch.* | *Pounds per sq. inch.* |
| Initial tension cylinder. | On measured stresses. | Before annealing | 50,818 | 59,350 | 31,316 | 15,578 |
| | | After annealing | 52,015 | .......... | .... ...... | 24,167 |
| | Theoretical maximum | | 70,000 | 71,652 | 47,650 | 24.634 |
| 3.2-inch field gun computed resistance | | | 38,250 | 41,030 | 21,730 | 13,995 |

*Conclusions.*—The graphic representation of the curves of stress, &c., on the accompanying plate 6 affords the best means of judging the results of the treatment of the forging. The close accordance of the measured and deduced curves of stress in the forging after treatment is not accidental, because, as previously stated, both curves depend upon the

equilibrium of the strains in the forging, and by construction they have one point in common. Their further coincidence, therefore, is evidence of the uniformity of results obtained by the treatment. The somewhat marked irregularity of the measured stresses near the bore in the breech section, both before and after annealing, has led to the construction of two deduced curves, of which it will be seen that the one marked No. 2 coincides most nearly with the measured curve. This No. 2 is based upon the measured compression on the second circle from the bore. There is apparent evidence in this section that the contractile force of the outer layers of metal in cooling was sufficient to over-compress the metal near the bore.

The strains engendered in all the sections by the interior cooling were apparently unnecessarily severe, and tended to produce too great a strain of tension toward the exterior for economy of resistance to interior pressure. Thus, in all of the sections, before annealing, it is seen that the curve of stress in action departs considerably from a horizontal line, and the limit of stress in action is reached first at or near the exterior surface.* Of the four, however, the breech middle section is exceptionally well disposed.

It is important to note the general resemblance of the measured curves of stress in the four sections as showing the regularity of the cooling treatment throughout the length of the forging, and that an inspection of the end sections would have disclosed the condition of initial tension in the whole forging. This forging, as shown on the drawing, had marked irregularity of sectional dimensions, yet the degree of initial tension in the several sections is in general proportional to the thickness of the section, and there is no general abnormal distortion of either thin or thick sections.

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*The position of the stress curve for the state of action is necessarily influenced by the selection of the value $\theta = 60{,}000$. If this value had been taken equal to 68,000, as given in the report of physical qualities of the metal before quoted, the stress curves in action would be considerably more elevated next the bore, and the estimated values of $P_0$ would be correspondingly increased.

This result was, however, to be expected, inasmuch as the theoretical curve of initial tension depends upon the thickness of wall in calibers, and the actual curve evidently obeys the same law.

This experiment leaves no room to doubt that initial tension strains of as great intensity as are desirable can be produced in a hollow forging by interior cooling, and if these strains should be more than needed they can be reduced by annealing. The effect of the subsequent annealing in the present case was beneficial, particularly in the muzzle section, which now shows the peculiarly interesting case of a resistance to interior pressure which closely approximates the possible maximum. The ordinates of the stress curve in action are all nearly equal and differ but little from the limit of 60,000 pounds.

Without disparagement to the built-up, hooped gun, which has proved to be excellent, it may be said that the apparent superiority of a gun made of a single forging, with initial tension produced by interior cooling, rests not only upon claims for reduced cost and increased longitudinal stiffness, but also for increased tangential strength in every section where the actual thickness of wall is insufficient in practice for the division of the built-up gun into as many as four layers, since this number of layers is in general required in that construction to enable the bore to be worked through the double limit of elastic movement.

Inasmuch as the walls of built-up, hooped guns below 10 or perhaps 8 inches caliber cannot be conveniently divided into four layers, and this division, moreover, can only be applied in the thicker portions (*i. e.*, the reinforce), the conclusion from the theoretical standpoint, at least, is that an equality of tangential strength will exist under the two modes of construction for the reinforce of guns of 8 or 10 inches caliber and upwards; but for guns of smaller caliber and for the chase portions of all guns the greater tangential strength will pertain to the single forging with initial tension produced by interior cooling.